Massively Parallel Graph Algorithms

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Warsaw, 07 October 2020

Research environment

Places (and people)

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- University of Wrocław (with Tomasz Jurdziński)

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- University of Wrocław (with Tomasz Jurdziński)
- ETH Zurich (with Mohsen Ghaffari)

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Financial support

- NCN grant no 2017/25/B/ST6/02010 (Tomasz' OPUS13 grant)
- NCN grant no 2019/32/T/ST6/00566 (my ETIUDA7 grant)
- FNP START scholarship (year 2020)

Models of computation

Modern Approach to Parallel Computing

Modern Approach to Parallel Computing

Computational model

- industry: MapReduce, Spark, ...
- TCS community: **M**assively **P**arallel **C**omputation (**MPC**) model

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Initial take on MapReduce

input data

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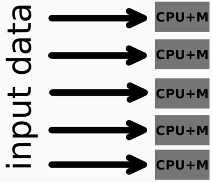

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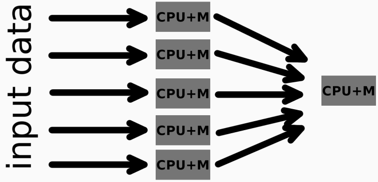

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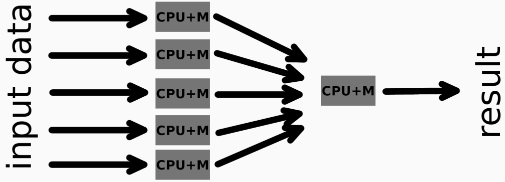

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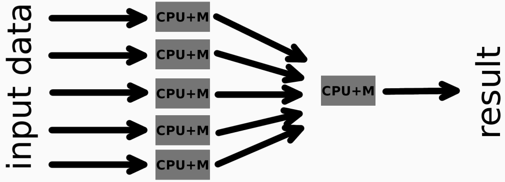

There are 2 kinds of problems:

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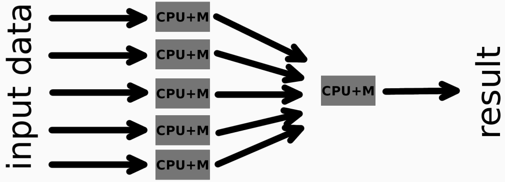

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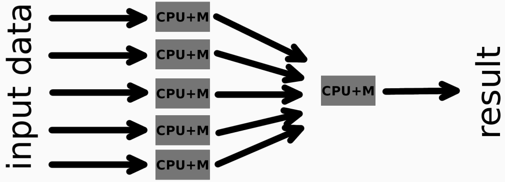

There are 2 kinds of problems:

- Easy: aggregative functions, e.g. sum, min, max, ...
- Non trivial: some graph problems, e.g. Minimum Spanning Tree

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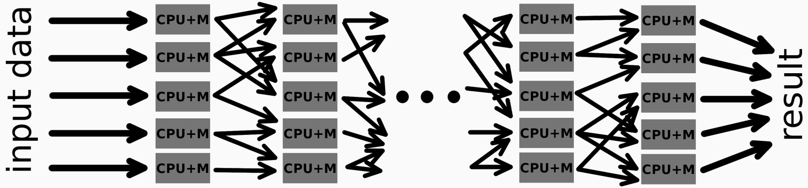

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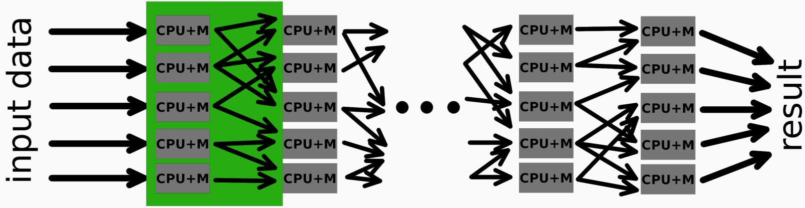

Computing in multiple rounds

- synchronous rounds

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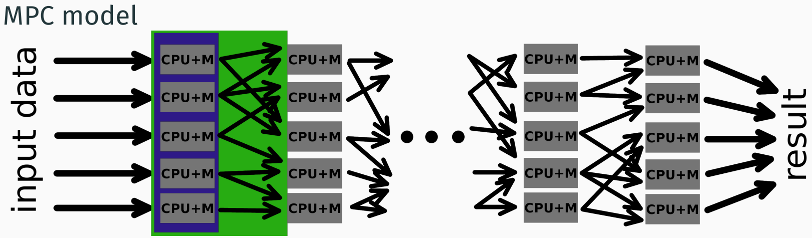

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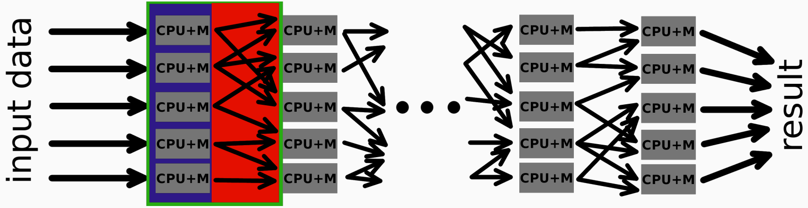

Computing in multiple rounds

- synchronous rounds
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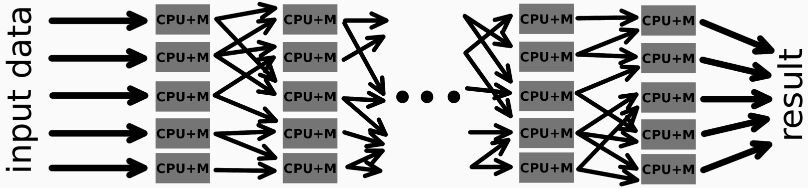

Parameters

- each machine gets input of size S / has communication limit S

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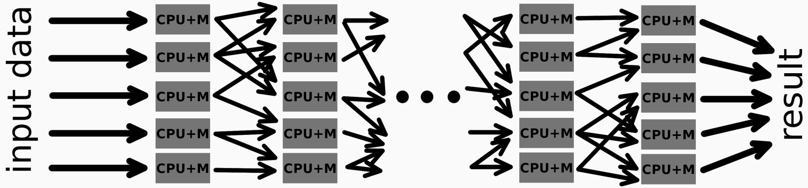

Parameters

- each machine gets input of size S / has communication limit S
- for input of size N, the number of machines is $\mathcal{O}(\frac{N}{S})$

Graph problems in the MPC model

Input / Output

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- graph with n vertices and m edges

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Remark: $m = \#$ of edges, $n = \#$ of vertices, $\delta =$ min degree

Congested Clique Model

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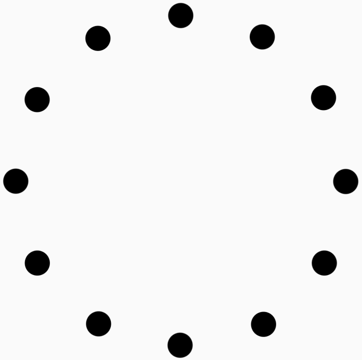
- multi party communication



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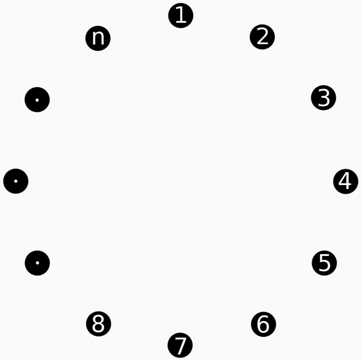

Congested Clique Model

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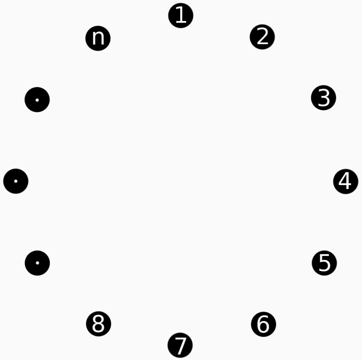

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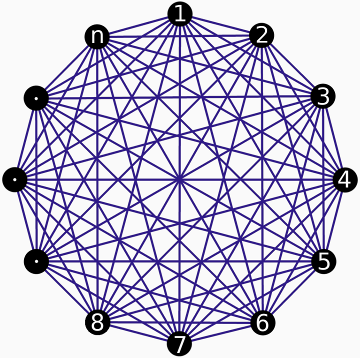

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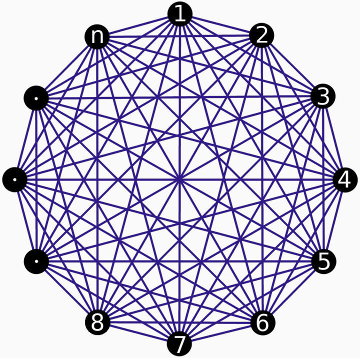

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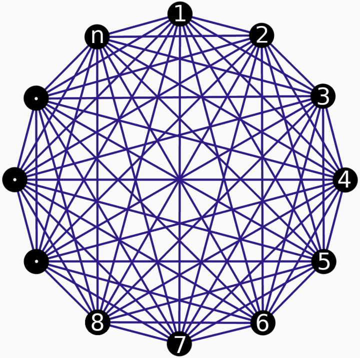

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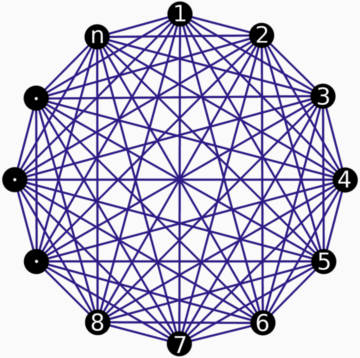

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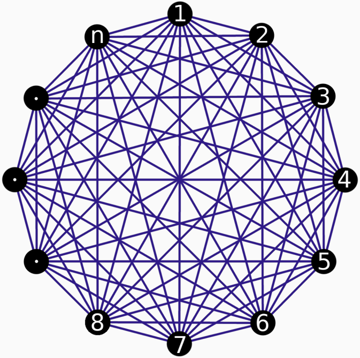

Remarks

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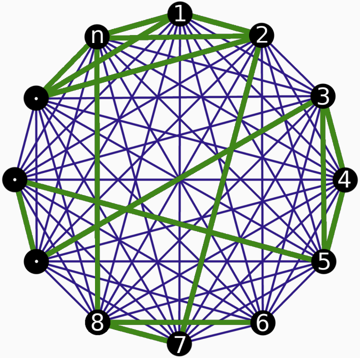

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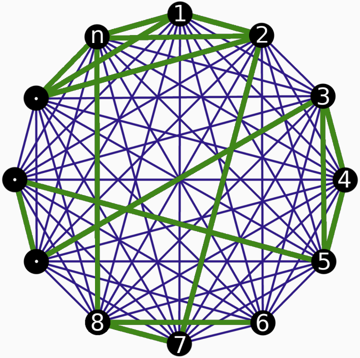

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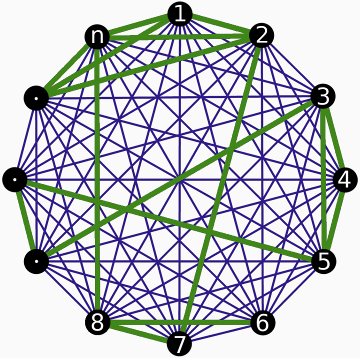

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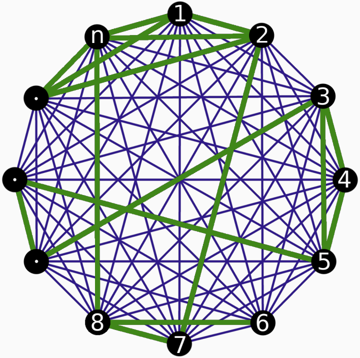

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- result: each player knows (part of) the output

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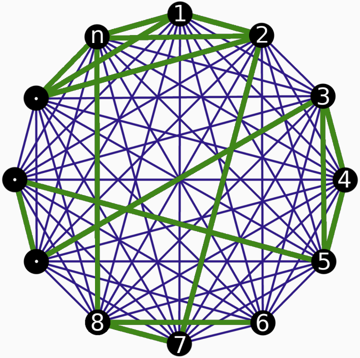

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Almost equivalent to MPC with $\mathcal{O}(n)$ local memory.

Output of the community (recent FOCS / STOC / SODA)

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*Dynamic Graph Algorithms with Batch Updates in the Massively Parallel Computation Model.(N., Onak; SODA'21)
*Walking Randomly, Massively, and Efficiently.(Łącki, Mitrović, Onak, Sankowski; STOC'20)
*Faster Algorithms for Edge Connectivity via Random 2-Out Contractions. (Ghaffari, N. Thorup, SODA'20)
*Parallel Batch-Dynamic Graphs: Constant Round Algorithms and Lower Bounds. (Durfee, Dhulipala, Kulkarni, Peng, Sawlani, Sun, SODA'20)
*Exponentially Faster Massively Parallel Maximal Matching. (Behnezhad, Hajiaghayi, Harris; FOCS'19)
*Conditional Hardness Results for Massively Parallel Computation from Distributed Lower Bounds. (Ghaffari, Kuhn, Uitto; FOCS'19)
*Near-Optimal Massively Parallel Graph Connectivity. (Behnezhad, Dhulipala, Esfandiari, Łącki, Mirrokni; FOCS'19)
*Massively parallel approximation algorithms for edit distance and longest common subsequence(Hajiaghayi , Seddighin, Sun, SODA'19)
*Sparsifying Distributed Algorithms with Ramifications in Massively Parallel Computation and Centralized Local Computation.(Ghaffari, Uitto; SODA'19)

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*Parallel Graph Connectivity in Log Diameter Rounds.(Andoni, Song, Stein, Wang, Zhong; FOCS'18)
*Round compression for parallel matching algorithms.(Czumaj, Łącki, Mądry, Mitrović, Onak, Sankowski; STOC'18)
*MST in O(1) Rounds of Congested Clique.(Jurdzinski, N.; SODA'18)
*Efficient massively parallel methods for dynamic programming.(Im, Moseley, Sun; STOC'17)
*A New Framework for Distributed Submodular Maximization.(Barbosa, Ene, Nguyên, Ward; FOCS'16)
*Randomized Composable Core-sets for Distributed Submodular Maximization. (Mirrokni, Zadimoghaddam; STOC'15)
*Parallel Algorithms for Geometric Graph Problems.(Andoni, Nikolov, Onak, Yaroslavtsev, STOC'14)
*A Model of Computation for MapReduce. (Karloff, Suri, Vassilvitskii, SODA'10)

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Including other papers makes the list *significantly* longer.

My research

Three main topics.

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1. Minimum Spanning Tree problem

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2. Minimum Cut problem

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1. Minimum Spanning Tree problem
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3. Dynamic Graph algorithms

My research

Minimum Spanning Tree problem

MST -- a problem with a long history of results

year	round complexity	det.?	authors; source

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2019	$\mathcal{O}(1)$	yes	N.; arxiv preprint

A sketch of a spanning forest algorithm

Hegeman et al. (PODC'15)

Instance of MST problem $\rightarrow$ several instances of Connected Components problem.

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Spanning Forest algorithm

1. Select $\mathcal{O}(n)$ edges (in a certain way).
2. Compute connected components on selected edges.
3. Partition the component graph into edge disjoint subgraphs, constructed in a certain way.
4. Sparsify all subgraphs, in parallel.

A sketch of a spanning forest algorithm

Hegeman et al. (PODC'15)

Instance of MST problem $\rightarrow$ several instances of Connected Components problem.

Spanning Forest algorithm

1. Select $\mathcal{O}(n)$ edges (in a certain way).
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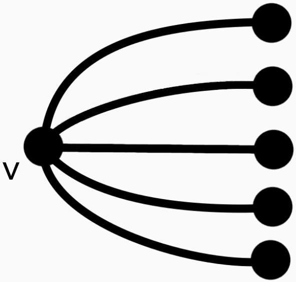

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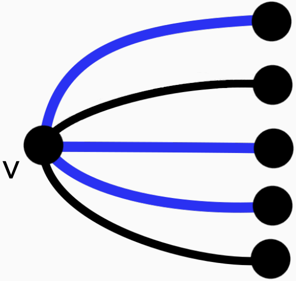

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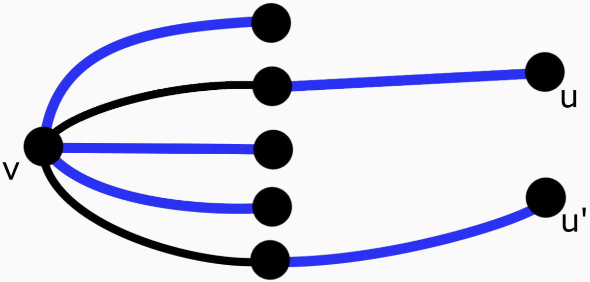

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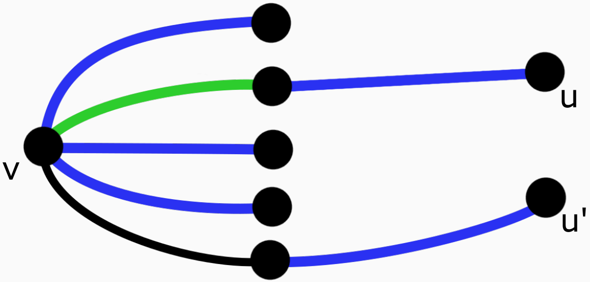

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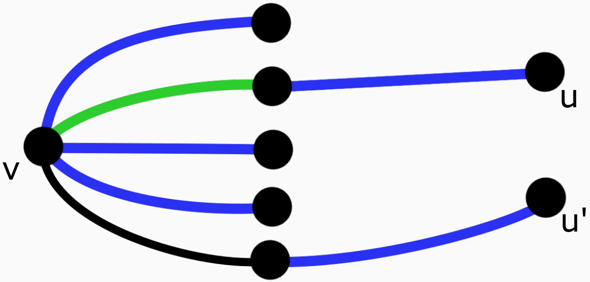

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$\Rightarrow$ v in component of size at least $deg(v) + 1$.,

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A sketch of the spanning forest algorithm

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Sparsification:

A sketch of the spanning forest algorithm

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A sparsification algorithm that reduces the number of edges to $\sqrt{mn}$, while preserving the edges of the minimum spanning forest.

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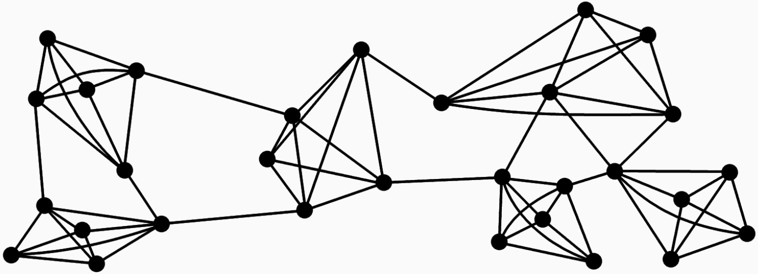
s.t. $\sum_i \sqrt{|V_i| \cdot |E_i|} = \mathcal{O}(n)$.

My research

Minimum Cut problem

2-out contraction - algorithm for simple graphs



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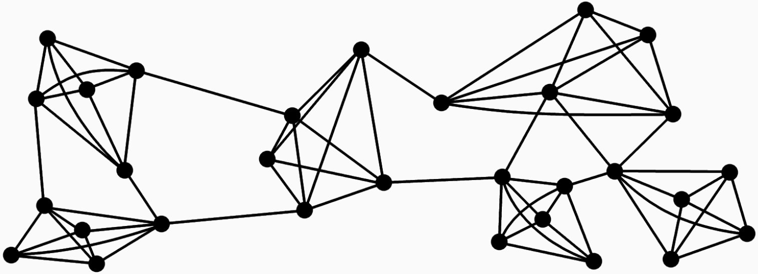

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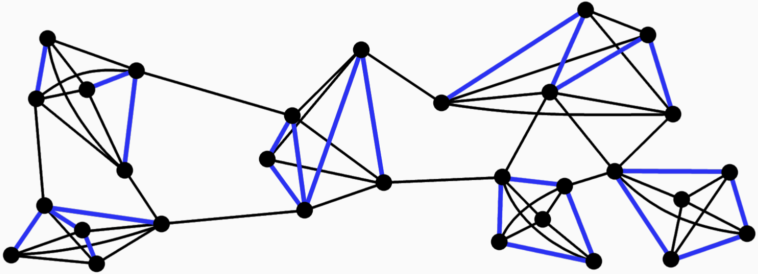

2-out contractions (Ghaffari, N., Thorup; SODA'20)

2-out contraction - algorithm for simple graphs

1. For each vertex mark 2 random incident edges.
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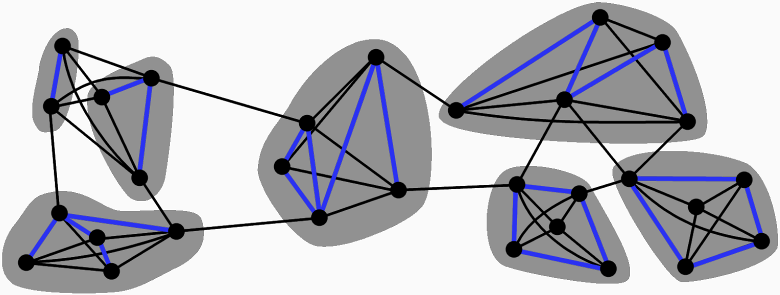

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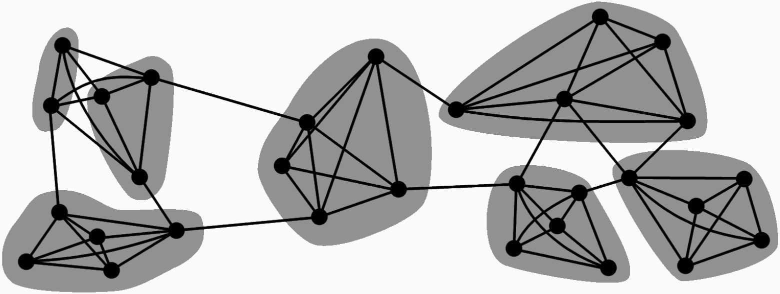

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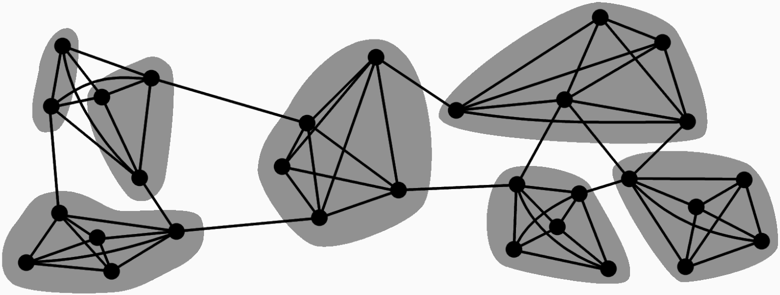

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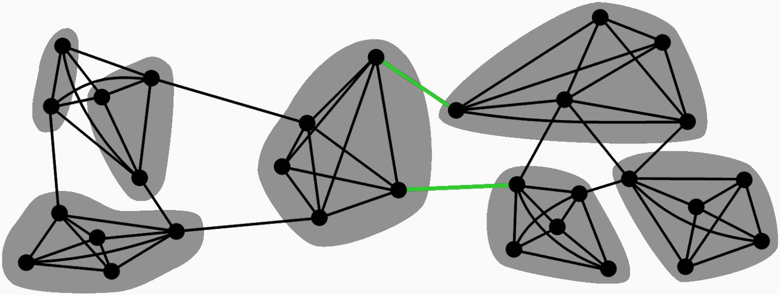

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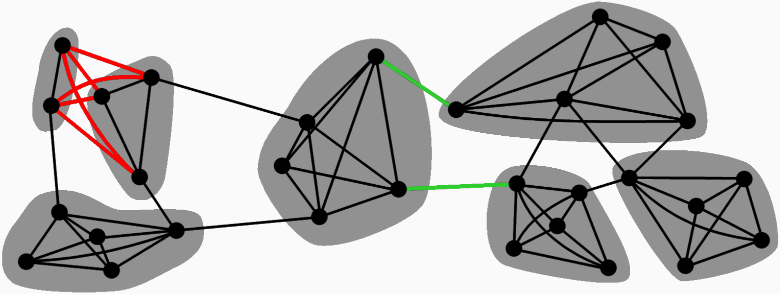

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MPC algorithms for weighted graphs (Ghaffari, N.; PODC'20)

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Sublinear local memory

$(2+\varepsilon)$-approximation in $\mathcal{O}(\log n \log\log n)$ rounds, modification of contraction based algorithm for PRAM (Karger, SODA'94).

My research

Dynamic Graph algorithms

Motivation

In practice:

Motivation

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- solving problem repeatedly

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- slightly changing data set

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- Goal: compute $S_1, S_2, \ldots,$ a sequence of solutions.

MPC: $S \in \Theta(n)$ vs $S \in \mathcal{O}(n^{1-\varepsilon})$

There is no proven separation between linear memory MPC and sublinear memory MPC. However, we know substantially better algorithms for $\mathcal{O}(n)$ memory regime.

	$S \in \mathcal{O}(n)$	$S \in \mathcal{O}(n^{1-\varepsilon})$
MST	$\mathcal{O}(1)$	$\mathcal{O}(\log n)$
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2-cycle Conjecture

Any algorithm that distinguish between single-cycle input and two-cycle input, with $\mathcal{O}(n^{1-\varepsilon})$ local memory, and $\mathrm{poly}(n)$ global memory requires $\Omega(\log n)$ rounds.

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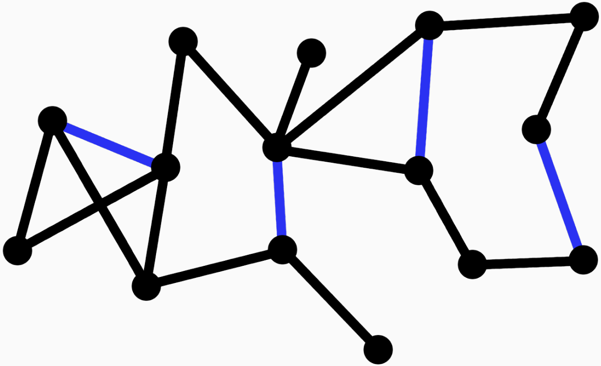
Example: Maximal Matching under batch updates

Input: Sequence of batch-updates, each adds / removes k edges.



Goal: Maintain a maximal matching; for each batch of updates to the graph, compute a batch of updates to the matching.

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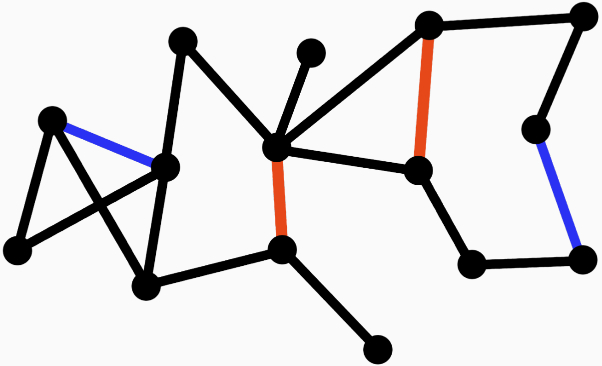

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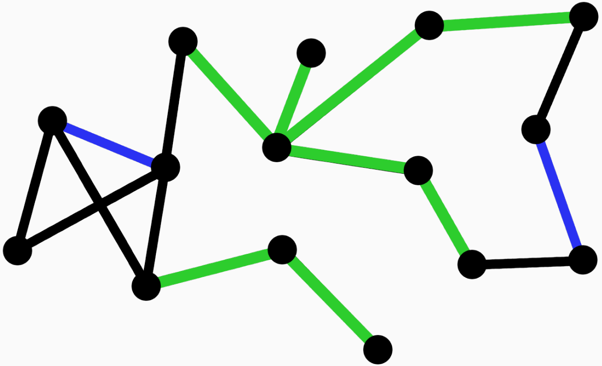

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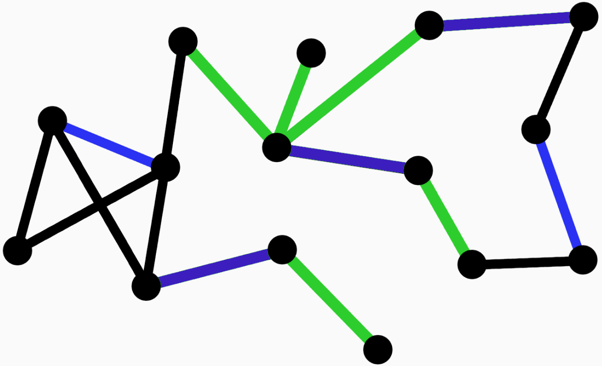

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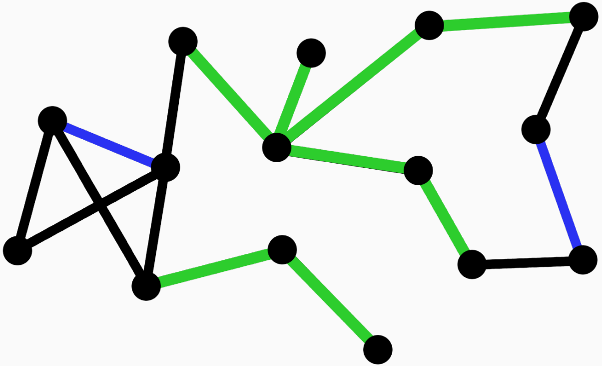

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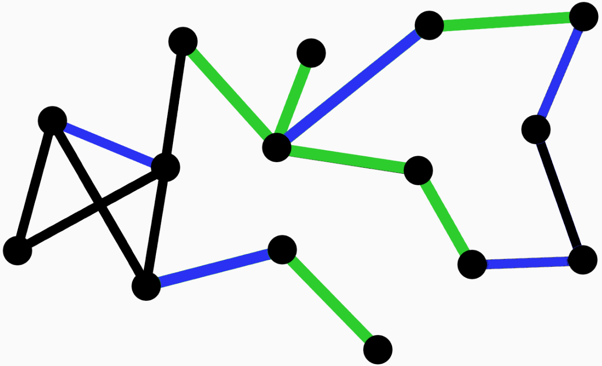

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Dynamic Graph Algorithms with Batch Updates in the MPC (N., Onak; SODA'21)

Our results (for $k \in \mathcal{O}(S)$)

Problem	upd. compl.	ini. compl.
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Better-than-static algorithms for $k \in \Theta(S^{1+\varepsilon'})$? Unlikely.

Thanks!

Thanks!

For the details:

- email me: knowicki@cs.uni.wroc.pl
- ask me in person