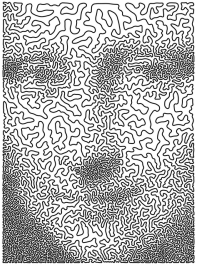
The Art of Incremental Improvements

Karol Węgrzycki



Context

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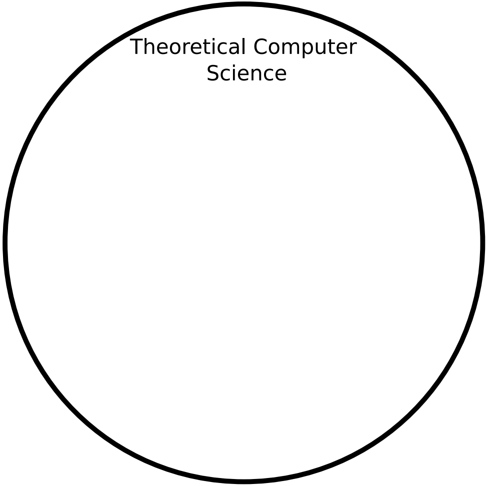

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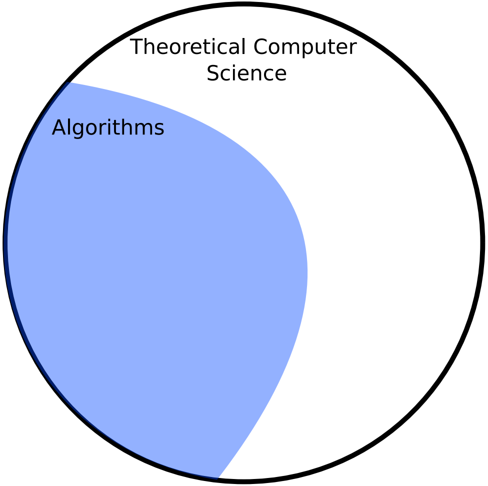

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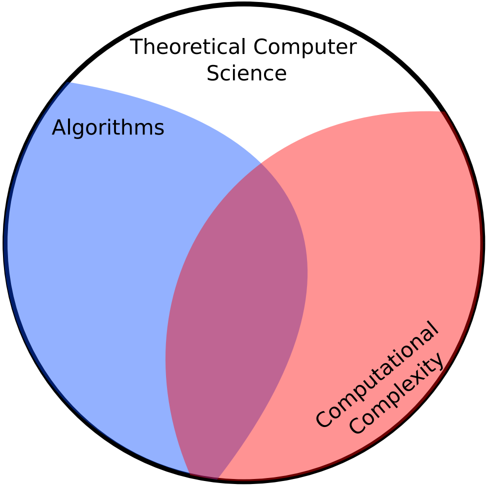
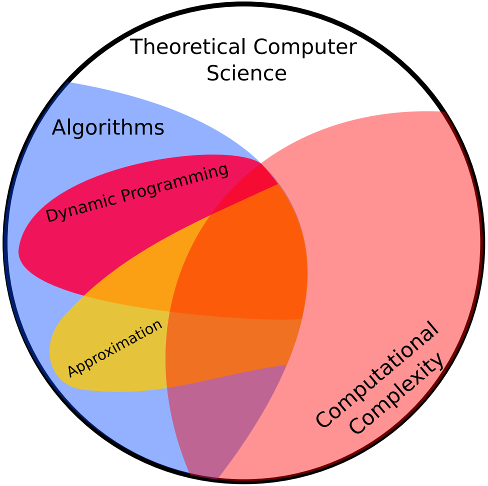

Theoretical Computer Science
Algorithms
Computational Complexity

Context


Theoretical Computer Science
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Dynamic Programming
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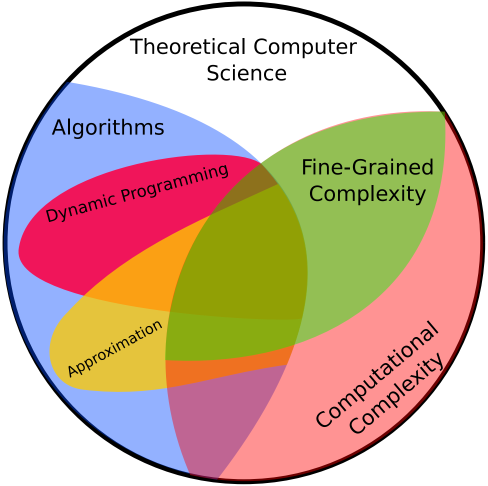

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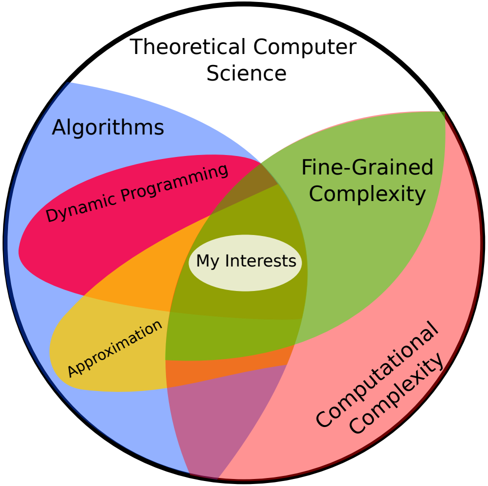
Theoretical Computer Science
Algorithms
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Fine-Grained Complexity
My Interests
Approximation
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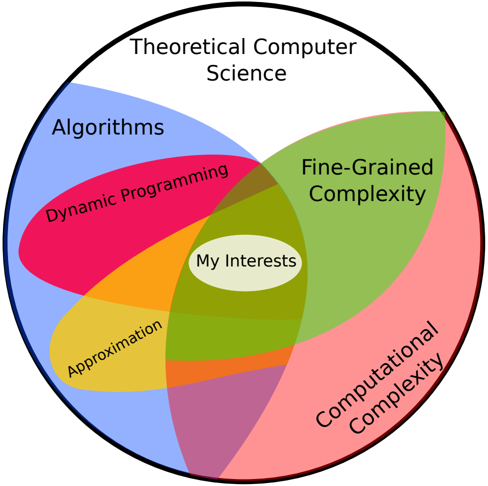

Goal: For *important* problem give:

- fast algorithm,
- argument that it cannot solved faster.

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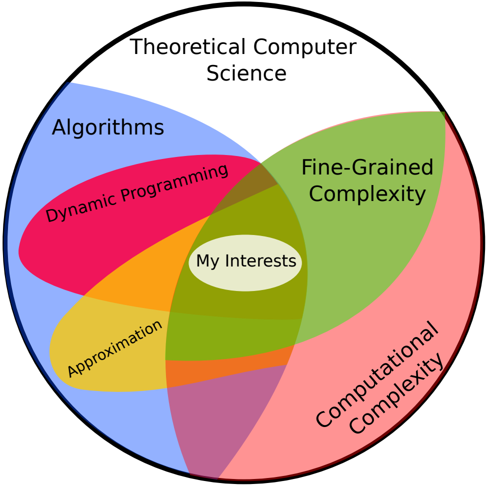

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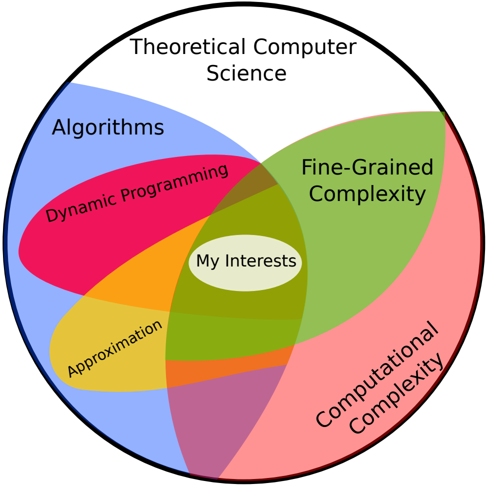

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Thank you: Antonios Antoniadis, Karl Bringmann, Jana Cslovjecsek, Marek Cygan, François Dross, Krzysztof Fleszar, Sándor Kisfaludi-Bak, Jędrzej Olkowski, Marcin Mucha, Jesper Nederlof, Marvin Künnemann, Andrzej Pacut, Jakub Pawlewicz, Michał Pilipczuk, Adam Polak, Lars Rohwedder, Mateusz Rychlewicz, Piotr Sankowski, Céline Swennenhuis, Adam Witkowski, Michał Włodarczyk, Anna Zych-Pawlewicz

Most of my talks look like this:

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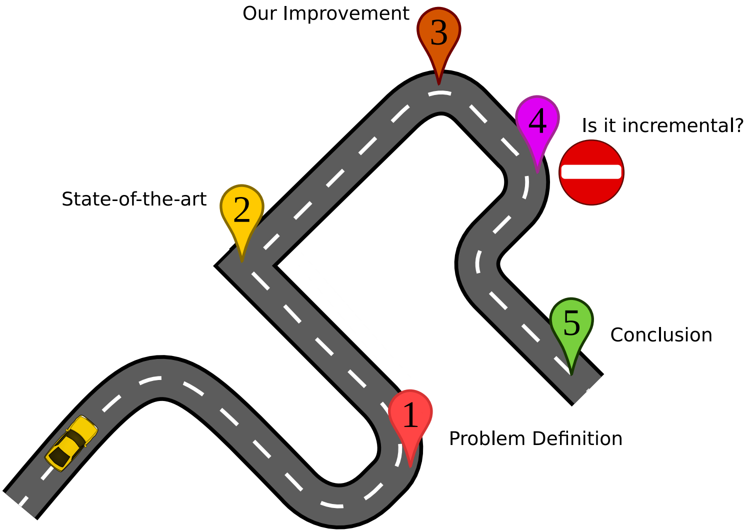
Plan for the talk



Part 1: Problem Definition

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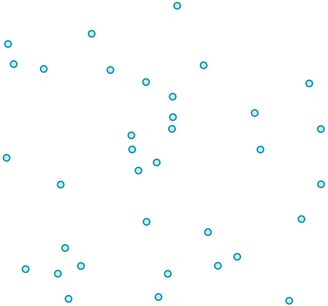
Euclidean Traveling Salesman Problem (Euclidean TSP): Given n points in $\mathbb{R}^2$, find the shortest roundtrip tour that contains all the points.



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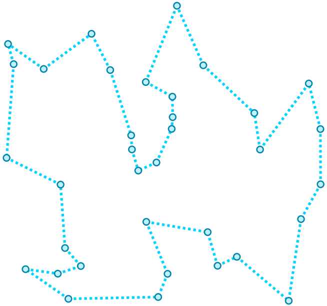

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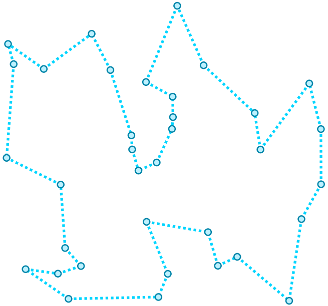

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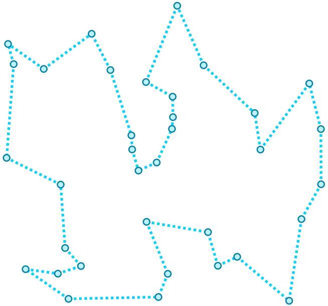

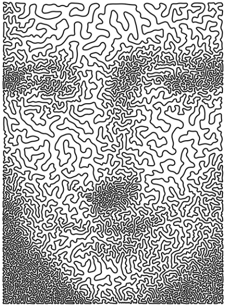

Source: Robert Bosch

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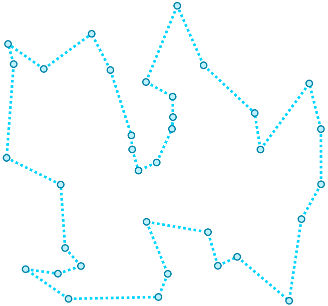

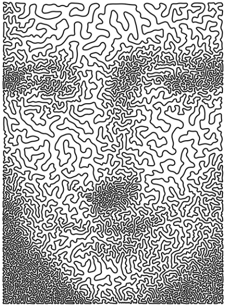

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Euclidean TSP is NP-hard [Papadimitriou 1977]. We need to settle for approximation

Part 2: State-of-the-art

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PTAS for Euclidean TSP: for any fixed $\varepsilon > 0$ find a tour that is at most $(1 + \varepsilon)$ times longer than optimal in $n^{\mathcal{O}(1)}$ time.

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History of PTASes for Euclidean TSP:

Arora (J.ACM 1998) $n(\log(n)/\varepsilon)^{\mathcal{O}(1/\varepsilon)}$ time

Mitchell (SICOMP 1999) $n^{\mathcal{O}(1/\varepsilon)}$ time

2010 Gödel prize winners!

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$(1/\varepsilon)^{\mathcal{O}(1/\varepsilon)} n \log n$ time

Bartal and Gottlieb (FOCS 2013) $2^{(1/\varepsilon)^{\mathcal{O}(1)}} n$ time

Complicated

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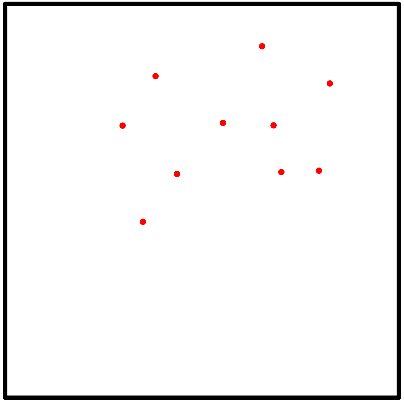
Our result: Simple $2^{\mathcal{O}(1/\varepsilon)} n \log n$ time algorithm (joint work with Sándor Kisfaludi-Bak and Jesper Nederlof)

Part 2: State-of-the-art

Arora's algorithm



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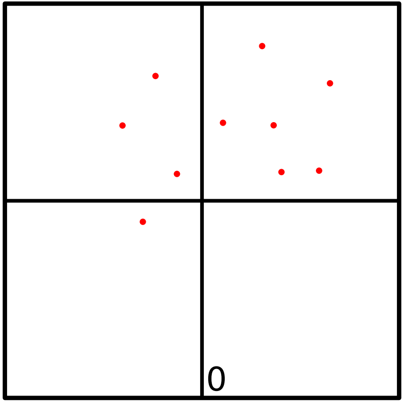

Algorithm:

1. Add randomly shifted quadtree

Part 2: State-of-the-art

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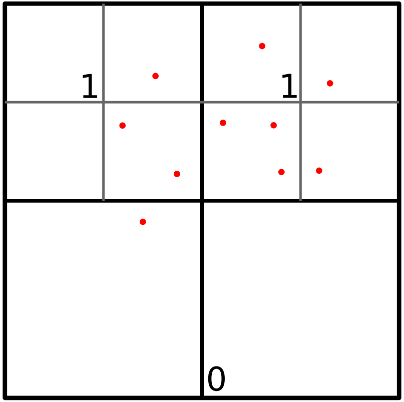

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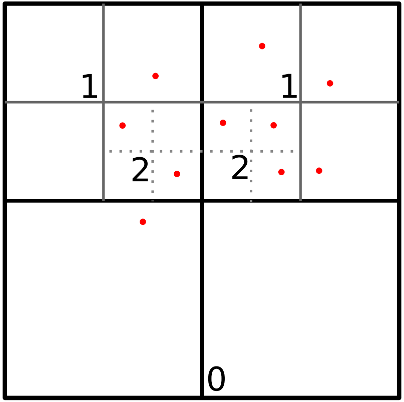

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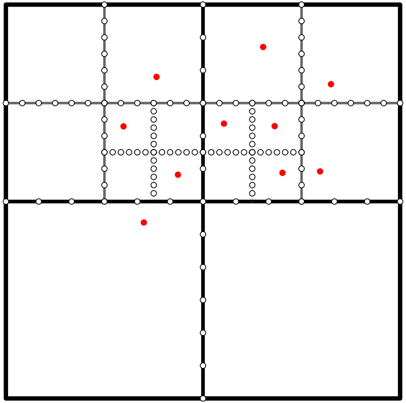

Algorithm:

1. Add randomly shifted quadtree
2. Add g equispaced portals for each cell

Part 2: State-of-the-art

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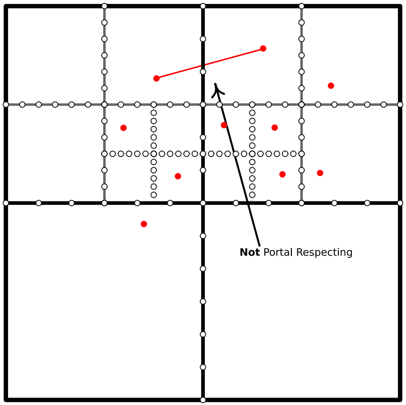

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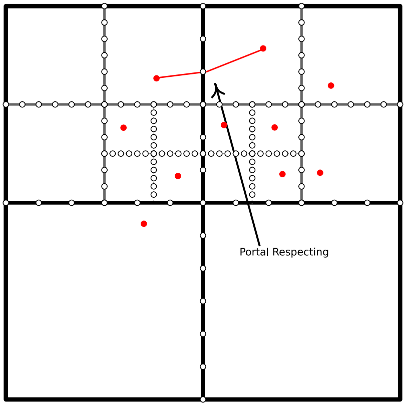

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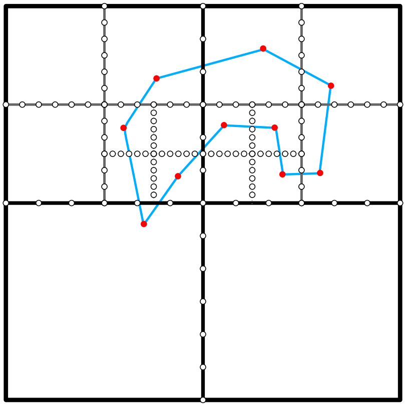

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Structure Theorem There is a portal respecting tour of expected tour length $\leq (1+\varepsilon)$OPT

Part 2: State-of-the-art

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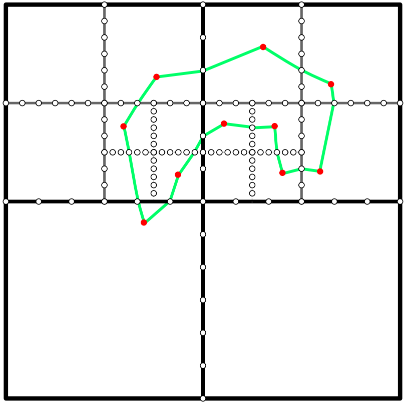

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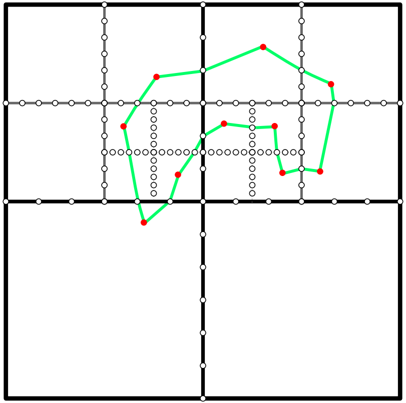

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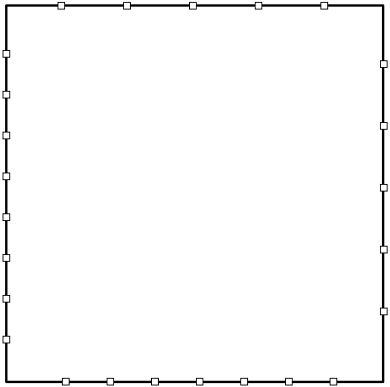

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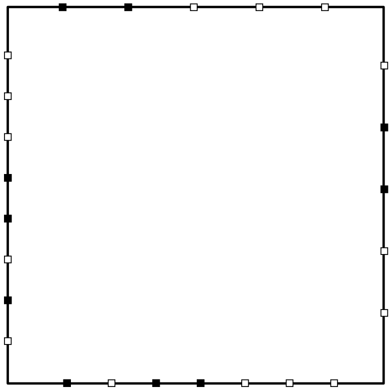

Guess selected portals out of $\mathcal{O}(\log(n)/\varepsilon)$

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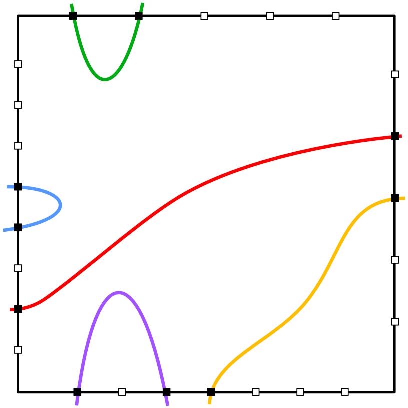

Guess noncrossing matching on selected portals

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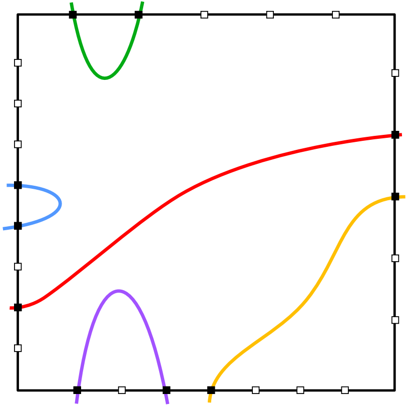

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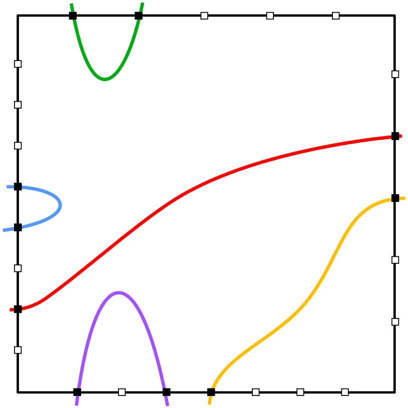

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Total runtime: $n^{\mathcal{O}(1)} \cdot 2^{\mathcal{O}(g)} = n^{\mathcal{O}(1/\varepsilon)}$

Part 3: Our Improvement

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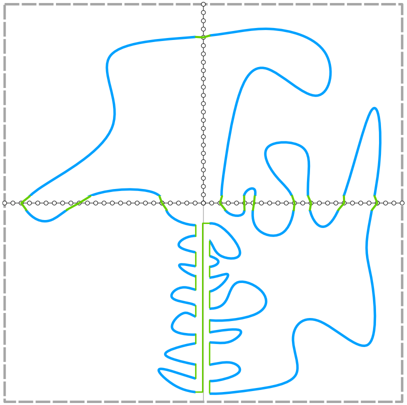

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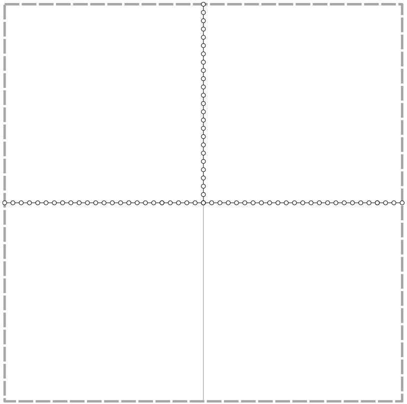

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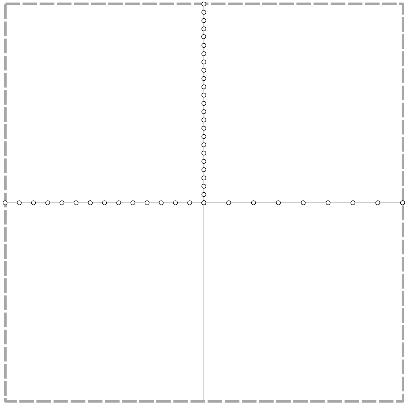

Main point: If tour uses $< 1/\varepsilon$ portals, then increase the granularity g

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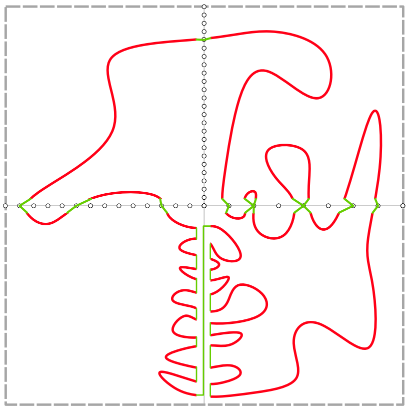

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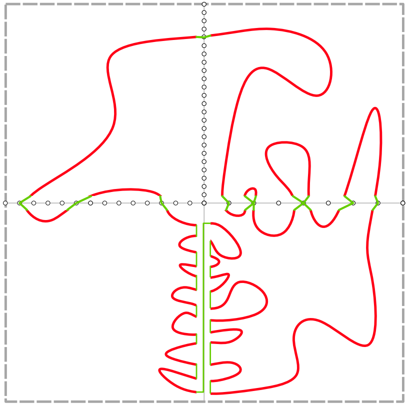

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Our idea For every cell pick $k \leq 1/\varepsilon$ portals from the set of $g := \frac{1}{\varepsilon^2 k}$ equally spaced portals

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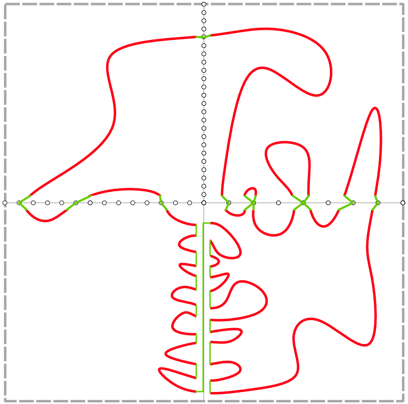

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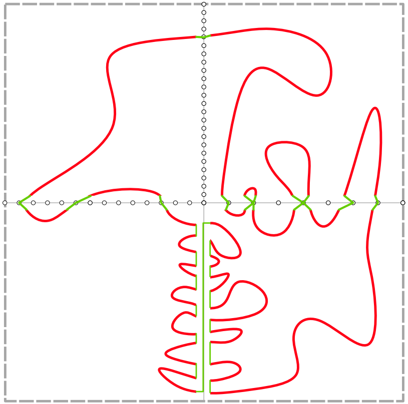
In the Dynamic Programming consider states where k portals from $g(k)$ equally spaced portals are used (for every $k \in \{2, \ldots, \varepsilon^{-1}\}$).

Part 3: Our Improvement



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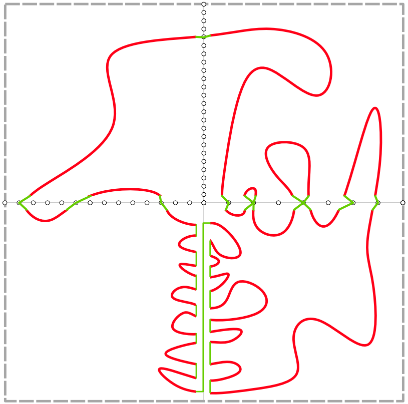

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Proof that this scheme works: Highly technical, check our paper

Part 4: Is it incremental improvement?

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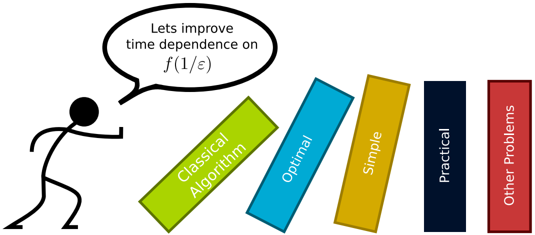

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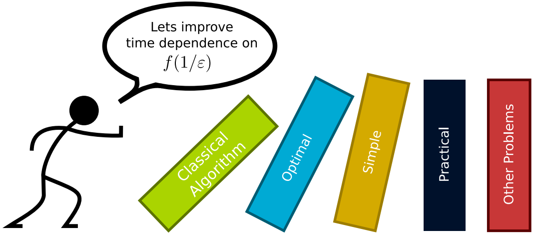

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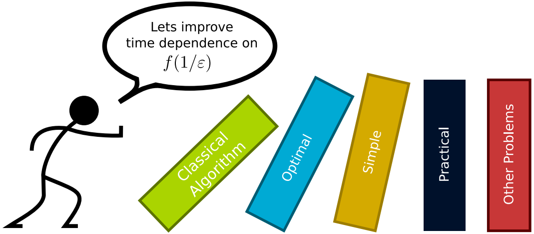

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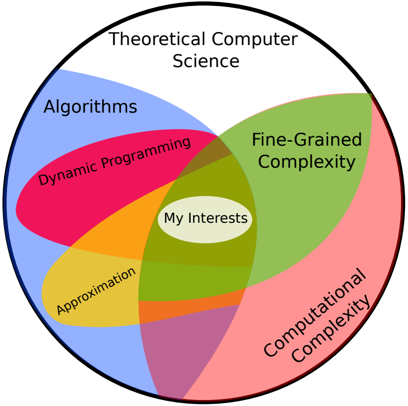
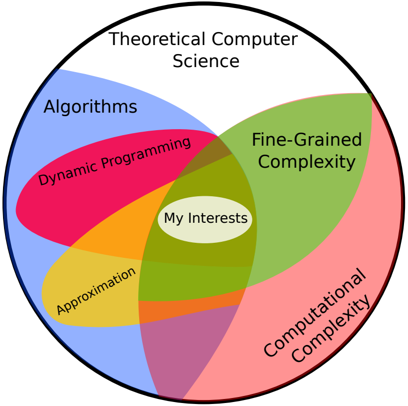
The point: This is the **final** improvement!

Conclusion



Theoretical Computer Science
Algorithms
Dynamic Programming
Fine-Grained Complexity
My Interests
Approximation
Computational Complexity

Conclusion



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My usual goal: settle the runtime complexity of fundamental problems

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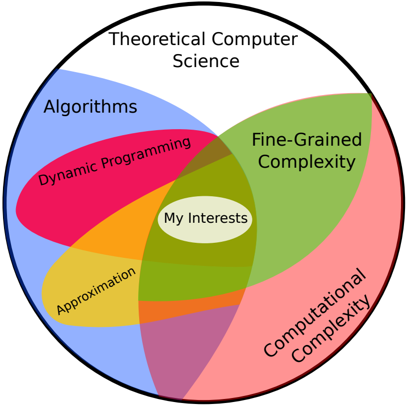

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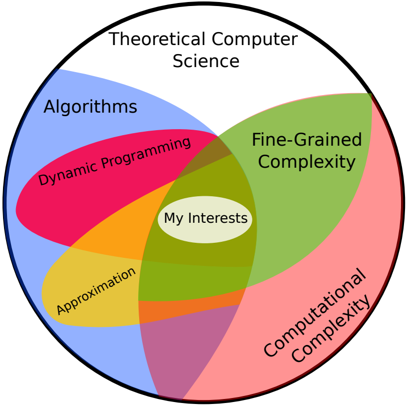

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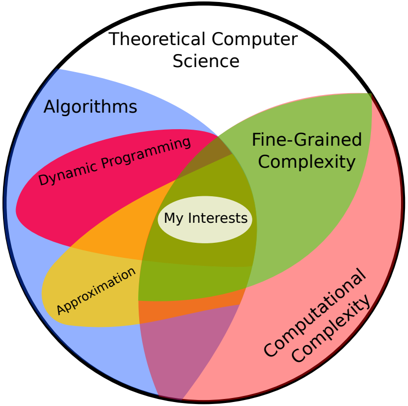

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Thank You!